

z — [112], Fig. 2. The matrix for transformation to the cubic axes is

$$\begin{array}{c|ccc} & x & y & z \\ \hline 1 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 2 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 3 & 0 & -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \end{array}$$

From equation (6) and the transformation matrix, we obtain

$$\begin{aligned} d\varepsilon_{11} &= d\varepsilon_{22} = d\varepsilon_{xx}/3, \quad d\varepsilon_{33} = -2d\varepsilon_{xx}/3, \\ d\varepsilon_{23} &= -d\varepsilon_{xx}/3, \quad d\varepsilon_{31} = d\varepsilon_{xx}/3, \quad d\varepsilon_{12} = 2d\varepsilon_{xx}/3. \end{aligned} \quad (9)$$

Equation (5) becomes

$$\begin{aligned} M &= \frac{1}{\tau} \left[-\frac{B}{3} + \frac{A}{3} - \frac{2F}{3} + \frac{2G}{3} + \frac{4H}{3} \right] \\ &= \frac{1}{3\tau} [-B + A - 2F + 2G + 4H]. \end{aligned} \quad (10)$$

The Bishop and Hill stress states that maximize the right side of (10) are Nos. 6 and 27, with $M = 4\sqrt{6}/3$.

For stress state 6 ($A = B = C = F = G = 0, H = \sqrt{6}\tau$), slip systems $a_1, -a_2, b_1, -b_2, -c_1, c_2, -d_1, d_2$ become active. And for state 27 ($A = B = C = 0, F = -\sqrt{6}\tau/2, G = H = \sqrt{6}\tau/2$), $-a_2, a_3, b_1, -b_3, -d_1$ and d_2 become active. Hence, the actual operating systems (which are common to both states) are $-a_2, b_1, -d_1$ and d_2 . It may be noted that systems $-d_1$ and d_2 are in cross-slip relationship with b_1 and $-a_2$, respectively.

A closer examination of this orientation, however, reveals the possibility of slip on systems $-a_2$ and b_1

alone. It may be shown that under these conditions,

$$\begin{aligned} d\varepsilon_{xx} &= -\frac{2}{\sqrt{6}}d\gamma, \quad d\varepsilon_{yy} = 0, \quad d\varepsilon_{zz} = \frac{2}{\sqrt{6}}d\gamma, \\ d\varepsilon_{yz} &= -\frac{\sqrt{3}}{6}d\gamma, \quad d\varepsilon_{zx} = 0, \quad d\varepsilon_{xy} = 0, \end{aligned} \quad (11)$$

where $d\gamma$ is the incremental shear each on slip systems $-a_2$ and b_1 . The only difference between equations (11) and (6) is in the shear strain term $d\varepsilon_{yz}$. However, since the present setup does not restrict $d\varepsilon_{yz}$ to zero, the deformation is expected to occur on $-a_2$ and b_1 alone if the total amount of shear $\Sigma|d\gamma_i|$ is less than that for the four slip systems case (Taylor's minimum shear principle). For slip on $-a_2$ and b_1 , we have from equation (11),

$$\Sigma|d\gamma_i| = 2|d\gamma| = \sqrt{6}d\varepsilon_{xx}, \quad (12)$$

and for slip on $-a_2, b_1, -d_1$ and d_2 ,

$$\Sigma|d\gamma_i| = Md\varepsilon_{xx} = \frac{4}{3}\sqrt{6}d\varepsilon_{xx}, \quad (13)$$

The former value is one-third less and hence we may expect slip on $-a_2$ and b_1 alone.

Similar calculations were carried out for five other orientations of interest. The results for all seven orientations are summarized in Table 2, together with those for the polycrystalline samples. Interestingly, the same operating slip systems as those listed in Table 2 were found earlier by a less rigorous method.⁽⁸⁾ As for the polycrystalline material, a value of $M = 1.44\sqrt{6}$ was used. It was derived by Hosford and Backofen⁽³⁾ from the von Mises yield criterion and the use of Taylor's factor of 3.06 for relating the tensile yield stress to the resolved shear stress for slip in a randomly oriented polycrystalline sample. Although the derivation was based on tensile testing under plane strain conditions, it may be shown that this value is

TABLE 2. Summary of analysis

Sample no.	Compression plane	Elongation direction	M	Slip systems selected	Equation (6) satisfied?
1	112	111	$3\sqrt{6}/2$	$-a_1, a_2, -c_3, d_3$	yes
2	110	112	$\sqrt{6}$	$-a_2, b_1$	no
			$4\sqrt{6}/3$	$-a_2, b_1, -d_1, d_2$	yes
3	110	001	$\sqrt{6}$	$a_1, -a_2, b_1, -b_2$	yes
4	110	110	$2\sqrt{6}$	$-a_1, a_2, -b_1, b_2, c_1, -c_2, d_1, -d_2$	yes
5	001	100	$\sqrt{6}$	a_2, b_2, c_2, d_2	yes
6	001	110	$\sqrt{6}$	$-c_1, c_2, -d_1, d_2$	yes
7	111	112	$3\sqrt{6}/2$	$-b_1, b_2, c_3, -d_3$	yes
8	polycrystal	#1	$1.44\sqrt{6}$	—	yes
9	polycrystal	#2	$1.44\sqrt{6}$	—	yes

Composition of all samples: 4% Mo- 17% Fe- 79% Ni by weight. Samples 8 and 9 were slowly cooled and quenched, respectively, after annealing at 1000°C; grain diameter ~ 0.04 mm.